New Experimental Limits on Non-Newtonian Forces in the Micrometer Range

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We report measurements of the short-range forces between two macroscopic gold-coated plates using a torsion pendulum. The force is measured for separations between 0.7 and 7 μm and is well described by a combination of the Casimir force, including the finite-temperature correction, and an electrostatic force due to patch potentials on the plate surfaces. We use our data to place constraints on the Yukawa-type ''new'' forces predicted by theories with extra dimensions. We establish a new best bound for force ranges 0.4–4 μm and, for forces mediated by gauge bosons propagating in $(4 + n)$ dimensions and coupling to the baryon number, extract a $(4 + n)$-dimensional Planck scale lower limit of $M_* > 70$ TeV.

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It is remarkable that two of the greatest successes of 20th century physics, general relativity and the standard model, appear to be fundamentally incompatible. Intense effort is devoted to searching for a framework that connects gravity to the rest of physics, and string theory, or M theory, is a candidate. There are still a number of outstanding problems; two of the most serious ones are the gauge hierarchy problem and the cosmological constant problem. Theoretical approaches have included proposals incorporating n extra spatial dimensions [1], predicting deviations from Newtonian gravity at submillimeter length scales. The rationale is to bring down the Planck scale from $M_P = 10^{19}$ GeV in 4 dimensions to the electroweak scale $M_* \approx 1$ TeV in $(4 + n)$ dimensions, thereby addressing the gauge hierarchy problem. In addition, in this scenario, gauge bosons that propagate in the bulk of the n extra dimensions but couple to the standard model baryon number can mediate forces that are a factor of $\approx 10(M_*/M_N)^2 \approx 10^7$ stronger than gravity; here $M_N \approx 1$ GeV is nucleon mass. These forces have the Yukawa exponential form, with the range given by the Compton wavelength of the boson, proportional to the inverse of its mass, whose natural scale is $\approx M_*^2/M_P$, diluted, exactly like the gravitational interaction, by the bulk $(4 + n)$-dimensional volume [2].

A large amount of experimental work has been done to search for such forces in a wide range of distance scales [3]. The Yukawa potential due to new interactions is typically taken to modify the gravitational inverse-square law:

$$V(r) = -G\frac{m_1 m_2}{r}(1 + \alpha e^{-r/\lambda}), \tag{1}$$

where G is the gravitational constant, m_1 and m_2 are the masses of the interacting particles, r is the distance between them, and the new interaction parameters are the strength α and the range λ. The strength α is constrained to be below unity for $\lambda > 56$ μm [4], but at shorter ranges the experimental limits are not as stringent [5–8]. The measurements at short ranges are complicated by the presence of the Casimir force [9,10], as well as the electrostatic forces due to surface patch potentials [11,12]. See [13] for a recent overview of tests of gravity at submillimeter ranges.

Recent measurements of the attractive force between two gold-coated flat and spherical plates for separations between 0.7 and 7 μm have improved our understanding of the Casimir and the electrostatic patch forces in this separation range and detected the thermal Casimir force [14]. We now use these measurements to place limits on new interactions in the micron range.

Our apparatus, which has been more fully described in Ref. [14], comprises a torsion pendulum suspended inside a vacuum chamber (pressure 5×10^{-7} torr) by a tungsten wire of 25 μm diameter and 2.5 cm length. The force to be measured is between the two glass plates, each coated with a 700 Å (optically thick) layer of gold evaporated on top of a 100 Å-thick layer of titanium. One is a flat plate mounted on one side of the pendulum, and the other is a spherical lens (radius of curvature $R = 15.6$ cm, as measured with a Micromap TM-570 interferometric microscope at the

Advanced Light Source Optical Metrology Laboratory [15,16] and found to vary by less than 2% over the surface of the lens), mounted on a Thorlabs T25 XYZ positioning stage, which, together with a piezoelectric transducer, is used to vary the plate separation d. The attractive force between the plates creates a torque on the pendulum body, which is counteracted by a pair of "compensator" electrodes on the opposite end of the pendulum. The voltage (on top of a constant offset voltage of 9 V, applied in order to linearize the response) that has to be applied to the compensator electrodes to keep the pendulum stationary is proportional to the force between the Casimir plates, with the calibration coefficient extracted from the measurements of the electrostatic force between the plates. Further details of the measurement technique can be found in Refs. [14,17].

The total force between the plates can be written as

$$F = F_{\mathrm{Casimir}} + F_{\mathrm{electric}} + F_{\mathrm{gravity}} + F_{\mathrm{new}}. \tag{2}$$

The gravitational (Newtonian) force between the plates, F_{gravity}, is very nearly a constant (≈ 20 pN) in the studied range of separations and is neglected in the analysis. F_{new} is the hypothetical new force, arising from the Yukawa potential in Eq. (1). The Casimir force between the spherical lens and the planar plate is calculated in the proximity force approximation (valid for $d \ll R$) as $F_{\mathrm{Casimir}} = 2\pi R E_{\mathrm{Casimir}}$, where E_{Casimir} is the Casimir interaction energy per unit area between two flat parallel plates separated by a distance d. The latter is computed by using the Lifshitz formalism with temperature $T = 300$ K and the gold optical permittivity data [18], extrapolated to zero frequency by using the Drude model with parameters $\omega_p = 7.54$ eV and $\gamma = 0.051$ eV [14].

The electrostatic force is given by the expression

$$F_{\mathrm{electric}} = \pi\epsilon_0 R\left[\frac{(V - V_m)^2}{d} + \frac{V_{\mathrm{rms}}^2}{d}\right], \tag{3}$$

where ϵ_0 is the permittivity of free space, V is the computer-controlled bias voltage applied between the plates, and the "minimizing potential" offset V_m is due to the contact potential difference of approximately 20 mV between the two plates, caused by the several solder contacts around the electrical loop connecting the two plates. Our measurements show that the minimizing potential $V_m(d)$ is nearly independent of separation in the $0.7\ \mu\mathrm{m} \le d \le 7\ \mu\mathrm{m}$ range (average variation is 0.2 mV). V_{rms} is a parameter characterizing the magnitude of the voltage fluctuations across the plates' surfaces, giving rise to a patch-potential electrostatic force given by the second term in brackets. Such voltage patches are always present even on chemically inert metal surfaces prepared in an ultraclean environment [19,20] and can be caused by spatial changes in surface crystalline structure, surface stresses, and adsorbed impurities or oxides. The exact form of the electrostatic patch force is determined by the patch voltage size distribution spectrum on the plates [11] and, in particular, by the relationship between three length scales: the typical patch size λ, the plate separation d, and the "effective interaction length" $r_{\mathrm{eff}} = \sqrt{Rd}$. In the limit $d \ll \lambda \ll r_{\mathrm{eff}}$, the patch force is well described by $\pi\epsilon_0 R V_{\mathrm{rms}}^2/d$ [12].

A further correction is needed to account for fluctuations in plate separation d [21]. The sources of these fluctuations are surface roughness of the plates and pendulum fluctuations, caused, for example, by apparatus vibrations. In addition to radius of curvature measurements, surface roughness measurements were performed with the Micromap TM-570 interferometric microscope, yielding an rms roughness of $S_q \approx 10$ nm for the curved plate and $S_q \approx 1$ nm for the flat plate. Vibration-caused fluctuations in d were measured by connecting an inductor in parallel with the Casimir plates and monitoring the resonance frequency of the resulting LC circuit; rms fluctuations of $\lesssim 40$ nm were recorded. In addition, a statistical error of ± 10 nm in the determination of d contributes in quadrature to the fluctuations mentioned above. We take the total rms plate separation fluctuation of $\delta = (40 \pm 20)$ nm. From the Taylor expansion of the Casimir force about the mean plate separation, we deduce that a correction term $F_C''\delta^2/2$ has to be added to the theoretical force when comparing with the experiment; the double prime denotes the second-order derivative with respect to d. In addition, since the same correction exists for the electrostatic force, the plate separation d extracted from the electrostatic calibration was corrected by a factor $1 + (\delta/d)^2$, and the electrostatic patch force V_{rms}^2/d was corrected by the same factor.

The data are well described by the Drude model, using the distance correction derived from auxiliary measurements as described above (no free parameters), together with a least-squares fit for two parameters, which are V_{rms} and an overall force offset. Given that only two well-understood fitting parameters are needed to fully describe our data, which span more than an order of magnitude in distance and more than 2 orders of magnitude in force, we are confident that, together with a $1/d$ patch-potential force, the finite-temperature Drude model provides the correct explanation of the Casimir force between Au surfaces. The reduced χ^2 of the fit is 1.04. Therefore, we can set bounds on additional forces that might be present, at a level of confidence based on the statistical fluctuations in the difference between the data and the corrected model. The force data, grouped into distance bins and averaged, are shown in Fig. 1, together with the best-fit line (solid red line) and the Casimir force (dashed blue line). The difference between the red and blue curves is due to the patch-potential $1/d$ force. The fit residuals are shown in the inset.

According to Eq. (2), these residuals can be used to place a limit on the hypothetical "new" force F_{new} between the plates. Integrating over the two gold- and

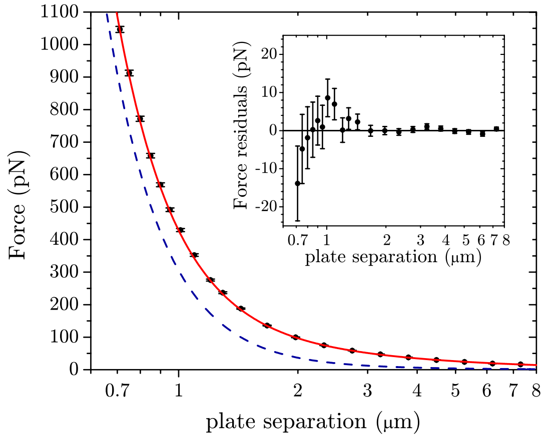

FIG. 1 (color online). The binned experimental short-range force between gold-coated plates. The error bars include contributions from statistical scatter, and uncertainties in the applied corrections, discussed in the text. The dashed blue line shows the theoretical Casimir force, calculated by using the Lifshitz formalism at 300 K, with the Drude model permittivity extrapolation to zero frequency. The red line shows the force, including the electrostatic patch-potential contribution, with two free fitting parameters, as described in the text. Inset: The force residuals, used to place constraints on the new short-range forces.

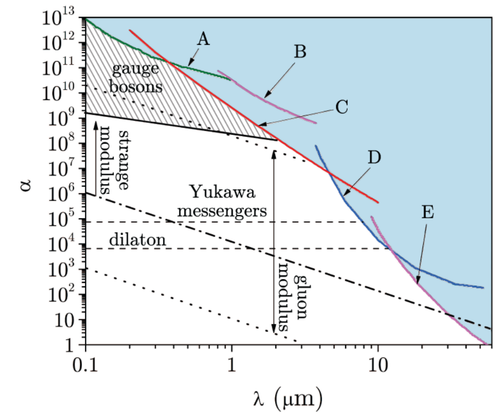

FIG. 2 (color online). Experimental upper limits on the Yukawa force strength α, together with some theoretical predictions. The area shaded in light blue is experimentally excluded. The curves labeled A–E correspond to results in Refs. [6,7], present work, and Refs. [4,5], respectively. The hatched area labeled "gauge bosons" is the parameter space for forces mediated by gauge bosons that propagate in $(4+n)$ dimensions and couple to the standard model baryon number.

titanium-coated plates gives the following approximate expression for the force:

$$F_{\rm new} = 4\pi^2 GR\alpha\lambda^3 e^{-d/\lambda}[\rho_{\rm Au} + (\rho_{\rm Ti} - \rho_{\rm Au})e^{-d_{\rm Au}/\lambda} + (\rho_{\rm g} - \rho_{\rm Ti})e^{-(d_{\rm Au}+d_{\rm Ti})/\lambda}]^2, \quad (4)$$

where $\rho_{\rm Au} = 19$ g/cm^3 is the gold density, $d_{\rm Au} = 700$ Å is the gold layer thickness, $\rho_{\rm Ti} = 4.5$ g/cm^3 is the Ti density, $d_{\rm Ti} = 100$ Å is the titanium layer thickness, and $\rho_{\rm g} = 2.6$ g/cm^3 is the substrate glass density. This expression is a good approximation to the exact form for the Yukawa force between the spherical lens and the flat plate, provided λ, $d_{\rm Au}$, and $d_{\rm Ti}$ are much less than the curved plate's radius of curvature R, the flat plate's thickness, and both plates' diameters. These conditions are satisfied very well in our experiment (for an exact expression for the force $F_{\rm new}$, not subject to these assumptions, see [22]). The obtained 95%-confidence limits on the new interaction strength α at each interaction range λ are shown in Fig. 2. The figure also shows limits obtained by other experimental groups, as well as some theoretical expectations. Our experiment achieves up to a factor of 30 improvement in the limit on the interaction strength α for $0.4\ \mu\text{m} < \lambda < 4\ \mu\text{m}$, compared to previous best limits [6].

Given the range of parameters α and λ that our experiment is most sensitive to, the most stringent limit we can place is on the $(4+n)$-dimensional Planck scale M_* in the presence of gauge bosons that propagate in $(4+n)$ dimensions and couple to the standard model baryon number (hatched region labeled "gauge bosons" in Fig. 2). Our data constrain the range of a hypothetical interaction mediated by such particles (i.e., their Compton wavelength) to be below 2 μm, which corresponds to the gauge particle mass of more than 0.5 eV. Assuming all the coupling parameters are on the order of unity, the natural scale for this mass is M_*^2/M_P, which means that the $(4+n)$-dimensional Planck scale is limited to $M_* > 70$ TeV. This is more stringent than the astrophysical limits, based on the PSR J09052+0755 neutron star heating from Kaluza-Klein graviton decay, for the case of 3 or more extra dimensions [23].

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